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Modelling Nuclear Weapon Effects in Wargaming Using Monte Carlo Simulations

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Abstract: The United States Army's interpretation of nuclear weapon effects needs change and modernization. Wargaming exercises are commonplace in today's military, however, despite the growing threat of non-strategic nuclear weapons (NSNW), little has been done to inform battlefield commanders on their true effects. Our research seeks to develop a tool for commanders to easily interpret quantifiable effects of a NSNW. Utilizing Monte Carlo simulation, we are developing a new methodology to analyze NSNW effects. Our model allows a commander to calculate the expected unit strength following a NSNW strike which will aid in their operational decision making ability. The Monte Carlo simulation method for analyzing nuclear effects offers a novel approach to account for variation while giving the commander an analytically interpretable output as descriptive statistics that avoids probabilities.

Keywords: Monte Carlo Simulation, Wargaming, Nuclear Effects, Non-strategic Nuclear Weapons

1. Introduction

A nuclear wargaming simulation analyzes the effects of a nuclear detonation on the battlefield. Wargaming is a tool that commanders use to develop plans and strategies to operate on the battlefield. The need for accurate nuclear wargames is critical as nation-state actors continue to increase their nonstrategic nuclear weapon (NSNW) arsenal (Woolf, 2022). For this study, NSNW are nuclear weapons with yield no greater than 30 kilotons, due to the limitation that NSNW must not lead to an escalatory nuclear response. These growing offensive capabilities pose new threats to modern battlefield commanders. Our model analyzes the detonation of a NSNW from an Inter-continental Ballistic Missile (ICBM) and its effect on friendly forces in the vicinity. The goal of our research is to determine the percent strength of a unit after detonation and quantify uncertainty in the descriptive statistics of mean and standard deviation for a battlefield commander.

This study focuses on the US Army's brigade combat teams, which are designed for defeating enemy forces, controlling terrain, securing populations, and preserving joint force freedom of action (FM 3-96, 2021). The two types of brigade combat teams we implemented are infantry brigade combat teams (IBCT) and armor brigade combat teams (ABCT). To accurately represent a battlefield, we analyzed the brigade, battalion, and company levels, subgroups of an IBCT and ABCT. Any company which did not have distinguishing combat equipment, (i.e., military intelligence or military police) was defined as a support company. All headquarters (HQ) companies were assumed to be the same size, with the same number of vehicles and personnel, to simplify the model without loss of generality. Also, all companies were assumed to act independently, stay organic, and not become attached to each other on the battlefield to maintain an individual footprint.

An IBCT is comprised of a HQ company, three infantry battalions, a cavalry squadron, a field artillery battalion, an engineer battalion, and a support battalion (FM 3-96, 2021). The infantry battalion contains a HQ company, three infantry (IN) companies, and a weapons company, which was simplified to be another infantry company. The cavalry squadron is made up of a HQ troop, two mounted troops, and a dismounted, infantry troop. The field artillery battalion has a HQ battery and three field artillery (FA) batteries. Engineer battalions contain multifunctional units which were combined and simplified into two engineer (EN) companies and two support (SPT) companies. Finally, a support battalion has six forward support (FWD SPT) companies and three support companies.

An ABCT is comprised of a HQ company, three combined arms battalions, a cavalry squadron, a field artillery battalion, an engineer battalion, and a support battalion (FM 3-96, 2021). We standardized the combined arms battalion to have a HQ company, two armor (AR) companies and a cavalry troop. The cavalry squadron contains a HQ troop, three cavalry (CAV) troops and an armor troop. The field artillery, engineer and support battalions are comprised of the same number of companies as they were in the IBCT.

These units employ varying amounts of specific equipment so to simplify the model, only five different components were represented across the IBCT and ABCT: soldier; small, wheeled vehicle (vehicle); large, wheeled vehicle (truck); light armor (LA) and tank. The specific breakdown of personnel and equipment by company can be referenced in Table 1 (FM 3-96, 2021; ATP 3-21.20, 2017; ATP 3-20.96, 2016; ATP 3-20.97, 2016; ATP 3-09.23, 2015).

Table 1. Number of unit types for each type of company.

	HQ	SPT	FWD SPT	IN	Mounted	IBCT FA	IBCT EN	AR	CAV	ABCT FA	ABCT EN
Soldier	80	100	100	120	120	100	100	60	100	60	100
Vehicle	19	8	6	3	8	21	10	3	17	6	10
Truck	1		8	1	1	4	10	1	1	1	10
LA									12	6	
Tank								12			2

Table 2 contains each unit type's vulnerability thresholds for blast, thermal, and initial radiation in interval notation (Glasstone & Dolan, 1977). The classifications of Fine, Injured, and Dead are assessed approximately one minute from the detonation of the NSNW. Fine indicates that the unit has received negligible effects on their ability to conduct the mission. Injured implies that the unit has taken some damage that limits their performance on the battlefield. Dead means that the unit is incapacitated and cannot continue the mission.

Table 2. Blast, thermal, and radiation vulnerability thresholds for each unit type.

	Blast (psi)			Thermal $\left(\frac{cal}{cm^2}\right)$			Radiation ($rems$)		
	Fine	Injured	Dead	Fine	Injured	Dead	Fine	Injured	Dead
Soldier	$[0,5]$	$(5,12)$	$[12,\infty)$	$[0,6]$	$(6,15)$	$[15,\infty)$	$[0,300]$	$(300,1000)$	$[1000,\infty)$
Vehicle	$[0,10]$	$(10,15)$	$[15,\infty)$	$[0,\infty)$			$[0,\infty)$		
Truck	$[0,10]$	$(10,15)$	$[15,\infty)$	$[0,\infty)$			$[0,\infty)$		
LA	$[0,10]$	$(10,15)$	$[15,\infty)$	$[0,\infty)$			$[0,\infty)$		
Tank	$[0,10]$	$(10,15)$	$[15,\infty)$	$[0,\infty)$			$[0,\infty)$		

2. Existing Simulations

The current leading modelling simulation in this field is the Defense Threat Reduction Agency's (DTRA) Mission Impacts of Nuclear Events Software (MINES). MINES relies on the probability of damage calculator (PDCalc) to perform their calculations. PDCalc, developed in 1974 using data from Hiroshima and Nagasaki by the Defense Intelligence Agency, uses an approximated distance-damage function to estimate the probability of destruction (Binninger, Castleberry, & McGrady, 1974). PDCalc's lognormal density function calculates the probability that a target will be destroyed. This metric leads to ambiguity when determining an exact count for percent strength after a nuclear strike because it only determines a probility of total destruction. Another limitation is that this model uses outdated and innacurate "damage σs" that act as the thresholds to determine the specified level of damage a target needs to receive to achieve an outcome (Binninger, Castleberry, & McGrady, 1974). Therefore, PDCalc accounts for the variation of NSNWs within the approximated probability function. We believe that a more accurate, modern methodology exists to give commanders more interpretable outputs and account for variation while using explicit damage thresholds.

3. Methodology

We use Monte Carlo simulation to overcome the shortfalls of PDCalc and give the commander an analytical output. Monte Carlo simulations rely on repeated random sampling and statistical analysis to compute the results (Raychaudhuri, 2008). The first phase of the simulation is identifying the input distributions. Next, we simulate multiple trials where each trial is one set of random numbers, consisting of one value for each of the input distributions, input through the deterministic model, to provide and store one set of output values. The final phase, output analysis, consists of calculating statistical metrics and displaying the values as a frequency histogram which provides the approximate shape of the probability density function. Once an output distribution is determined, convergence theory analysis is performed to find the error of our sample mean from the true population mean.

3.1 Model Inputs

3.1.1 NSNW Emplacement

The emplacement of the NSNW is the first randomized input to the Monte Carlo simulation. This variable is sampled from the three dimensional multivariate normal distribution. Equation 1 is the multivariate normal distribution (Tong, 2012):

$$f(x) = \frac{1}{2\pi^{n/2}\,|\Sigma|^{1/2}}\, e^{\left(-\frac{1}{2}(x-\mu)^T\,\Sigma^{-1}\,(x-\mu)\right)} \tag{1}$$

Where,

$$n = 3, \mu = [\mu_x \quad \mu_y \quad \mu_z], \text{ and } \Sigma = \begin{bmatrix} \sigma_x & 0 & 0 \\ 0 & \sigma_y & 0 \\ 0 & 0 & \sigma_z \end{bmatrix}.$$

The vector μ is the predicted impact locations in the x, y, and z directions. μ_x and μ_y are input by the commander for where they expect the NSNW to impact. μ_z is 213.36 meters, the optimal height of burst (HOB) for an airburst that minimizes fallout of a NSNW with a yield of 30 KT (Glasstone & Dolan, 1977).

The covariance matrix, Σ, is the error associated with the predicted impact location of the NSNW. Error in the emplacement is attributed to guidance errors (inertial guidance system) and non-guidance errors (winds, atmospheric density, geophysical uncertainties, and engine cut-off anomalies) (Moran, 1966). Circular error probable (CEP) is a metric that accounts for this variation in the x and y directions and is defined as "the radius of a circle within which 50% of the impact points will fall if the distribution of impact points is assumed to be normally distributed" (Binninger, Castleberry, & McGrady, 1974). A common ICBM delivery system has an estimated CEP of .2 nautical miles and when converted to meters, has a CEP of 370 meters. To calculate σ_x and σ_y, we use equation 2 (Binninger, Castleberry, & McGrady, 1974):

$$\sigma = CEP\,\frac{1}{\sqrt{\ln(4)}} \tag{2}$$

From equation 2, σ_x and σ_y are both 314.24 meters. Also, ICBMs are equipped with a height of burst sensor that utilizes terrain contour matching (TERCOM), but the information regarding the variance in these systems is not publicly released (Siouris, 2004). Since TERCOM operates under electromagnetic countermeasure conditions, day/night, and all weather, we assume a small standard deviation of 10 meters in σ_z.

3.1.2 Unit Emplacement

The emplacement of the friendly forces is the second randomized input to the Monte Carlo simulation. The friendly forces are uniformly distributed within company sized areas at specified locations. Mission dictates how company sized units and above maneuver on a battlefield so for our model, we choose to recursively emplace units in a wedge formation (V-shaped formation) with 1595.77 meter diagonal spacings from a specified center (ADP 3-90, 2019). The unit's size is determined by the commander's inputs, first specifying which unit they are commanding (IBCT or ABCT), and then selecting which level (brigade, battalion, or company). Reference Table 1 for the number of each unit type within each uniformly distributed circle. The x and y coordinates of each individual unit are found by solving equation 3 (Corner, 2018):

$$(x_{center} + r\cos(\theta), y_{center} + r\sin(\theta)) \tag{3}$$

We first find θ, a random angle between 0 and 2π, to set the angular position of the individual unit using equation 4:

$$\theta = 2\pi\, rand(0,1) \tag{4}$$

Next, we find r, a random distance of the individual unit from the origin of the circle:

$$r = R\sqrt{rand(0,1)} \tag{5}$$

The company sized unit's circle has a radius of $R = 564.189$ m, the maximum distance that the unit will be spread out. To find R, we solve for the radius of a circle equal to 100,000 m^2, which is from the area of a 1000 m by 1000 m grid square, the normal area in which a company sized element operates. This process, using equations 3, 4 and 5, is repeated based on the number of each unit type that will be within each company's circle to find the x and y coordinates of each individual unit.

3.2 Model

Taking both randomized inputs, we calculate the distance between the NSNW location and individual unit locations. This distance is input into three different distance damage functions to find the nuclear effects of blast, thermal, and radiation that each individual unit will receive. These equations of blast, thermal, and radiation calculate the pressure in psi, thermal radiation in $\frac{cal}{cm^2}$, and initial radiation in rads, respectively (Glasstone & Dolan, 1977). A nuclear weapon detonation is comprised of 50% blast energy, 35% thermal energy, 10% delayed radiation energy, and 5% initial radiation energy. In our model, blast accounts for only static overpressure (the crushing of objects) and negates dynamic overpressure (wind and the movement of objects). Also, our model only factors in initial radiation energy, radiation produced within one minute of the explosion. Due to classification, the distance damage equations are not publicly releasable. The values for the nuclear effects that each individual unit will receive are compared to the unit vulnerability thresholds in Table 2 to determine which individual units are Fine, Injured, and Dead. Table 3 contains weights for each individual unit type based on their relative importance to the mission, changed at the commander's discretion. The final step calculates the percent strength of the total unit, which takes the individual units' statuses and weights them according to the values in table 3.

Table 3. Weights for each individual unit type for the total unit's percent strength calculation.

	Soldier	Vehicle	Truck	LA	Tank
Fine	1	5	5	15	30
Injured	.5	2.5	2.5	7.5	15
Dead	0	0	0	0	0

3.3 Model Outputs

Each trial outputs the percent strength for the unit after a NSNW strike. After n trials, the following statistical metrics are calculated: mean, standard deviation (SD), maximum (max) and minimum (min). Also, the outputs are displayed as a frequency histogram to show the approximate shape of the probability density function.

4. Monte Carlo Simulation Results

Figure 1 displays a single iteration from the Monte Carlo simulation of the infantry battalion (IN BN) and the IBCT units. In these figures, the blue point represents the actual NSNW impact location with $\mu = [0 \quad 0 \quad 213.36]$, centered directly above the first unit in the IN BN and centered above the first unit of the middle BN in the ICBT. The IN BN took considerable damage but still had units in the company farthest from the blast that were within the threshold of Fine. Similarly, trucks and/or vehicles with larger vulnerability thresholds than that of soldiers were Fine in the two adjacent companies. The IBCT shows consistent trends with the impact on units closer to the NSNW impact location. A significant portion of the IBCT was also Fine showing that a NSNW will not eliminate every asset that the commander has in their unit.

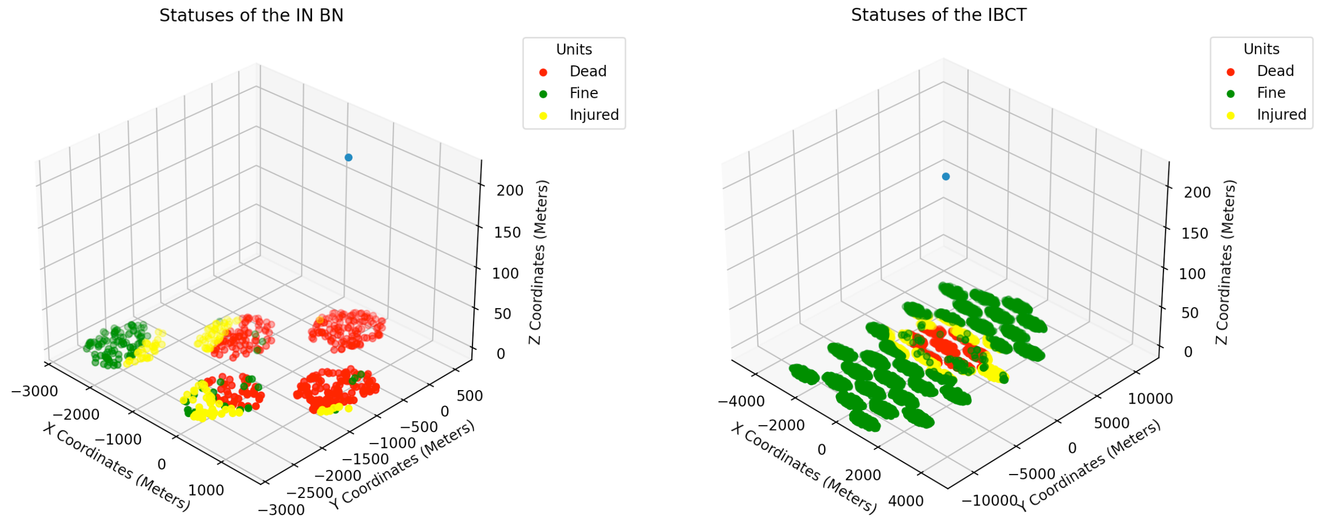

Figure 1. Plots of unit statuses for an IN BN (left) and an IBCT (right) from 1 iteration of the Monte Carlo simulation.

The Monte Carlo simulation, $n = 1000$, was executed at each level of command for both an ICBT and ACBT and the results are summarized in Table 4. The results in Table 4 confirm findings in Figure 1 from a single iteration, where units that are Dead are associated with a decreased percent strength and Fine relates to an increased percent strength. At the company level, IN COs lost every soldier with the occasional truck and/or vehicle surviving on the outskirts. The AR COs' results were similar with the possibility of tanks surviving, greatly increasing the unit's strength, but still leaving the unit with detrimental percent strength for follow on missions. At the battalion level, IN BNs were more affected than Combined Arms BNs with a mean percent strength 16.43 percent less. IBCTs and ABCTs are expected to lose 14.98 percent (*SD* = .14) and 11.42 percent (*SD* = .13), of their total strength after the impact of a NSNW, respectively. These results are crucial for the battlefield commander to assess their unit's strength and prepare for future operations. Figure 2 is a frequency histogram of the percent strength outputs from each trial and provides the approximate shape of the probability density function. The percent strengths appear normally distributed. Both the IN BN and the IBCT distributions are unimodal, have minimal skew, and are centered around their respective means. Finally, to validate our findings and determine the percent error of our sample mean, $\bar{x}$, from the true population mean, μ, we conducted convergence analysis on the IBCT and ABCT results. Using equation 6, we calculate the confidence interval for the true population mean (Oberle, 2015):

$$\mu = \bar{x} \pm \left(z_{\alpha/2} \frac{100\, s}{\bar{x}\sqrt{n}} \right) \tag{6}$$

The 95% confidence intervals of the true population means are (85.01, 85.03) for an IBCT and (88.57, 88.59) for an ABCT.

Table 4. Summary statistics for different unit's percent strengths from the Monte Carlo simulation.

	Company Level		Battalion Level		Brigade Level	
	IN CO	AR CO	IN BN	Combined Arms BN	IBCT	ABCT
Mean	.25%	1.63%	41.5%	57.93%	85.02%	88.58%
SD	.66%	2.32%	.69%	.74%	.14%	.13%
Max	3.57%	17.61%	43.85%	61.5%	85.43%	89.15%
Min	0%	0%	39.46%	56.82%	84.45%	88.21%

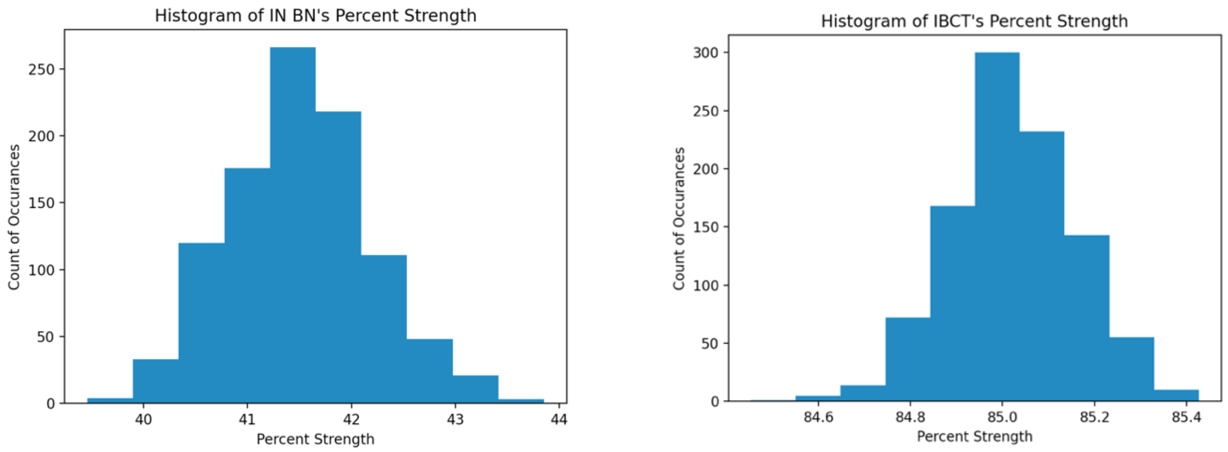

Figure 2. Frequency histograms of percent strength for an IN BN (left) and an IBCT (right).

5. Conclusion

The Monte Carlo simulation method for analyzing nuclear effects offers a novel approach to account for variation in nuclear effects. Adopting this methodology will greatly enhance the effectiveness of wargaming simulations because it avoids the output of the probability of total destruction in favor of expected values. Using the statistical metrics of mean and standard deviation will show commanders their expected strengths and associated variation following a NSNW strike. The ability to accurately determine attrition rates will enable commanders to develop multiple courses of actions (COA) for follow-on operations. Future work in this field should focus on incorporating terrain into the model to analyze real-world battlefields, providing more accuracy outside of a one-dimensional plane. Another limitation that can be addressed is that our model does not incorporate shielding effects (i.e., buildings and trees). Also, adding dynamic overpressure and delayed radiation effects will provide more accurate assessments. Finally, our model can be used to develop doctrine for large unit maneuver to minimize the effects of a NSNW. Commanders utilizing our methodology will be better informed of the threats they face from NSNWs, enabling them to minimize risk, react effectively in combat, and ultimately be more prepared on the nuclear battlefield.

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